3 x 3 + 1

Advice for Cambridge Maths Students

Michael Ng

Brief Introduction

- Part IA, IB, II - 2017-2020
  - Mix of pure, applied and applicable
- Part III - 2020-
  - Focus on Probability and Statistics
- Moving on to industry

- Pianist, racket sports, running, hanging out with friends!

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Concise points. Three sections, three points each. And an extra.

‘What I’d tell my past self.’

- Resources
- Revision
- Future

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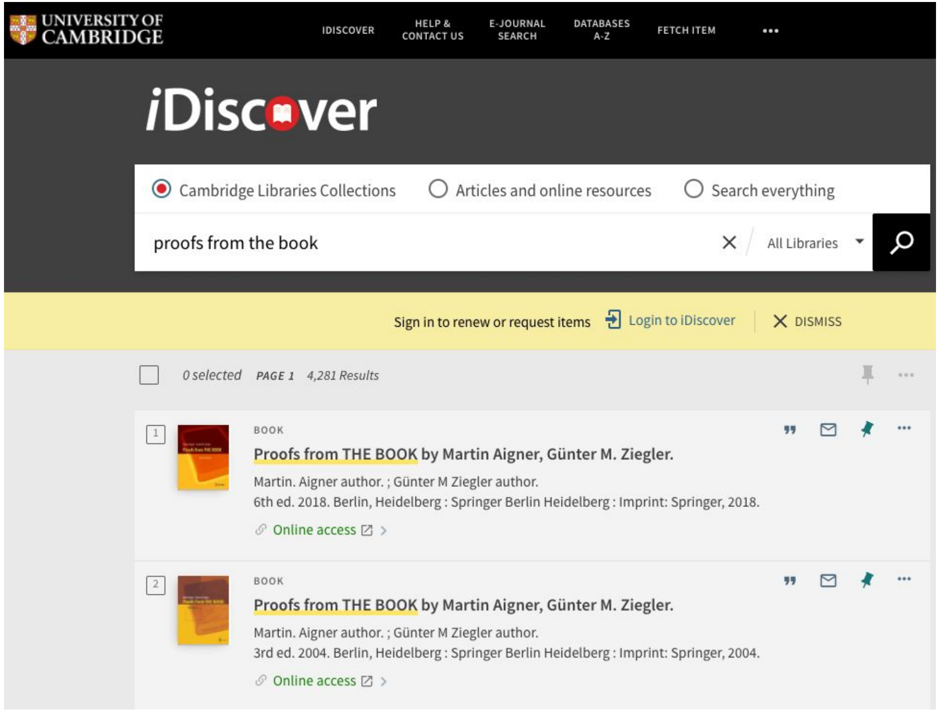
(**1** + 1 + 1) x 3 + 1

- **Resources**
    - **iDiscover** - *references* in course descriptions (many free)



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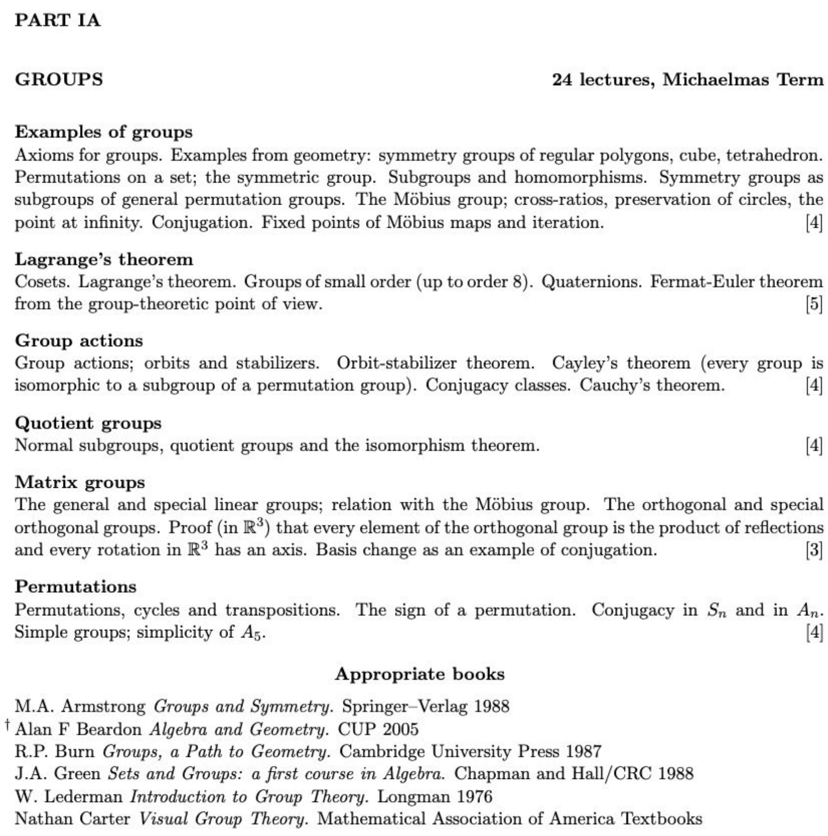
PART IA

GROUPS **24 lectures, Michaelmas Term**

Examples of groups
Axioms for groups. Examples from geometry: symmetry groups of regular polygons, cube, tetrahedron. Permutations on a set; the symmetric group. Subgroups and homomorphisms. Symmetry groups as subgroups of general permutation groups. The Möbius group; cross-ratios, preservation of circles, the point at infinity. Conjugation. Fixed points of Möbius maps and iteration. [4]

Lagrange's theorem
Cosets. Lagrange's theorem. Groups of small order (up to order 8). Quaternions. Fermat-Euler theorem from the group-theoretic point of view. [5]

Group actions
Group actions; orbits and stabilizers. Orbit-stabilizer theorem. Cayley's theorem (every group is isomorphic to a subgroup of a permutation group). Conjugacy classes. Cauchy's theorem. [4]

Quotient groups
Normal subgroups, quotient groups and the isomorphism theorem. [4]

Matrix groups
The general and special linear groups; relation with the Möbius group. The orthogonal and special orthogonal groups. Proof (in $\mathbb{R}^3$) that every element of the orthogonal group is the product of reflections and every rotation in $\mathbb{R}^3$ has an axis. Basis change as an example of conjugation. [3]

Permutations
Permutations, cycles and transpositions. The sign of a permutation. Conjugacy in S_n and in A_n. Simple groups; simplicity of A_5. [4]

Appropriate books

M.A. Armstrong *Groups and Symmetry.* Springer–Verlag 1988
† Alan F Beardon *Algebra and Geometry.* CUP 2005
R.P. Burn *Groups, a Path to Geometry.* Cambridge University Press 1987
J.A. Green *Sets and Groups: a first course in Algebra.* Chapman and Hall/CRC 1988
W. Lederman *Introduction to Group Theory.* Longman 1976
Nathan Carter *Visual Group Theory.* Mathematical Association of America Textbooks

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  - **Study Group** - *regular* discussions with friends

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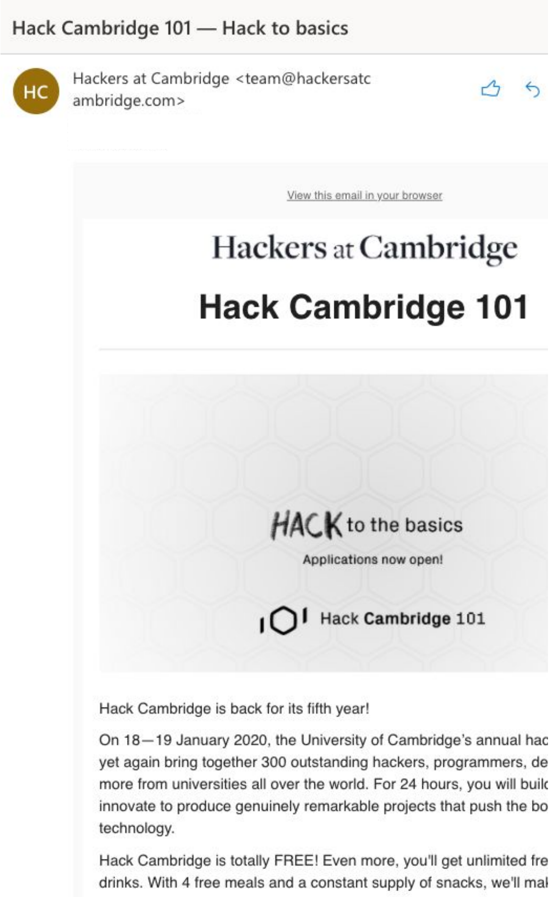
  - **Study Group** - *regular* discussions with friends
  - **Mailing Lists** - maths societies, coding competitions, career insight days (more later)


Hack Cambridge 101 — Hack to basics

HC Hackers at Cambridge <team@hackersatcambridge.com>

View this email in your browser

Hackers at Cambridge

Hack Cambridge 101

Hack Cambridge is back for its fifth year!

On 18—19 January 2020, the University of Cambridge's annual hac
yet again bring together 300 outstanding hackers, programmers, de
more from universities all over the world. For 24 hours, you will builc
innovate to produce genuinely remarkable projects that push the bo
technology.

Hack Cambridge is totally FREE! Even more, you'll get unlimited fre
drinks. With 4 free meals and a constant supply of snacks, we'll mal

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- **Revision**
  - **Content > Past Papers** - Memory emphasis. Be efficient! Focus on the non-bookwork parts when doing past papers.

2016

Paper 3, Section I

2D Groups

State and prove Lagrange's theorem.

2015

Paper 3, Section II

7D Groups

(a) State and prove Lagrange's theorem.

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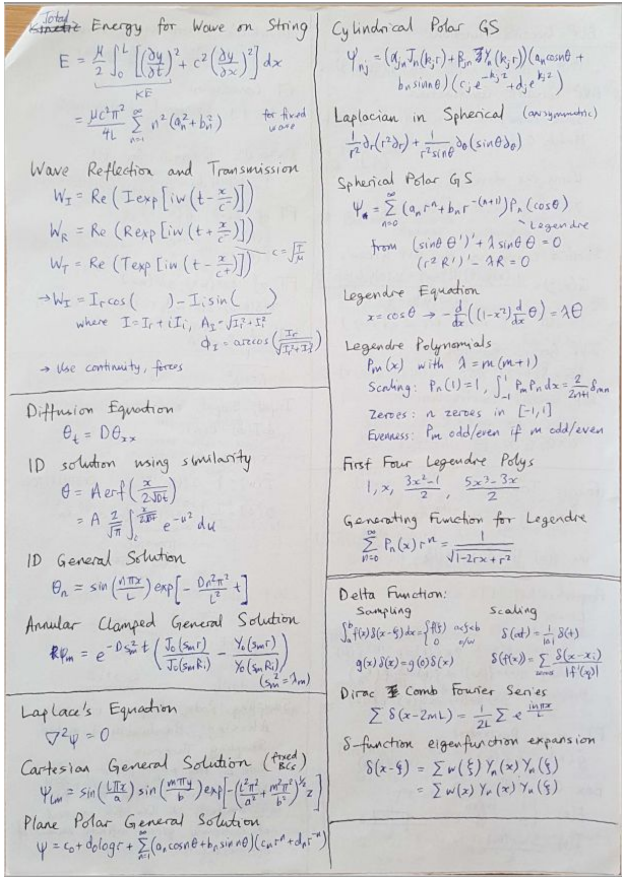
  - **Flashcards** - *essential*. Active recall. E.g. paper/Anki.



~~Kinetic~~ Total Energy for Wave on String

$$E = \frac{\mu}{2}\int_0^L \left[\left(\frac{\partial y}{\partial t}\right)^2 + c^2\left(\frac{\partial y}{\partial x}\right)^2\right] dx$$

KE

$$= \frac{\mu c^2 \pi^2}{4L}\sum_{n=1}^{\infty} n^2 (a_n^2 + b_n^2)$$ for fixed wave

Wave Reflection and Transmission

$$W_I = \mathrm{Re}\left(I \exp\left[i\omega\left(t - \frac{x}{c^-}\right)\right]\right)$$

$$W_R = \mathrm{Re}\left(R \exp\left[i\omega\left(t + \frac{x}{c^-}\right)\right]\right)$$

$$W_T = \mathrm{Re}\left(T \exp\left[i\omega\left(t - \frac{x}{c^+}\right)\right]\right)$$ $c = \sqrt{\frac{T}{\mu}}$

$\rightarrow W_I = I_r \cos(\quad) - I_i \sin(\quad)$

where $I = I_r + iI_i$, $A_I = \sqrt{I_r^2 + I_i^2}$

$\phi_I = \arccos\left(\frac{I_r}{\sqrt{I_r^2 + I_i^2}}\right)$

→ Use continuity, forces

Diffusion Equation

$$\theta_t = D\theta_{xx}$$

1D solution using similarity

$$\theta = A\,\mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = A\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{Dt}}} e^{-u^2} du$$

1D General Solution

$$\theta_n = \sin\left(\frac{n\pi x}{L}\right)\exp\left[-\frac{Dn^2\pi^2}{L^2}t\right]$$

Annular Clamped General Solution

$$\psi_m = e^{-Ds_m^2 t}\left(\frac{J_0(s_m r)}{J_0(s_m R_i)} - \frac{Y_0(s_m r)}{Y_0(s_m R_i)}\right) \quad (s_m^2 = \lambda_m)$$

Laplace's Equation

$$\nabla^2\psi = 0$$

Cartesian General Solution (fixed BCs)

$$\psi_{lm} = \sin\left(\frac{l\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\exp\left[-\left(\frac{l^2\pi^2}{a^2} + \frac{m^2\pi^2}{b^2}\right)^{1/2} z\right]$$

Plane Polar General Solution

$$\psi = c_0 + d_0 \log r + \sum_{n=1}^{\infty}(a_n \cos n\theta + b_n \sin n\theta)(c_n r^n + d_n r^{-n})$$

Cylindrical Polar GS

$$\psi_{nj} = (\alpha_{jn} J_n(k_j r) + \beta_{jn} Y_n(k_j r))(a_n \cos n\theta + b_n \sin n\theta)(c_j e^{-k_j z} + d_j e^{k_j z})$$

Laplacian in Spherical (axisymmetric)

$$\frac{1}{r^2}\partial_r(r^2\partial_r) + \frac{1}{r^2\sin\theta}\partial_\theta(\sin\theta\,\partial_\theta)$$

Spherical Polar GS

$$\psi = \sum_{n=0}^{\infty}(a_n r^n + b_n r^{-(n+1)})P_n(\cos\theta)$$ ← Legendre

from $(\sin\theta\,\Theta')' + \lambda\sin\theta\,\Theta = 0$

$(r^2 R')' - \lambda R = 0$

Legendre Equation

$$x = \cos\theta \rightarrow -\frac{d}{dx}\left((1-x^2)\frac{d}{dx}\Theta\right) = \lambda\Theta$$

Legendre Polynomials

$P_m(x)$ with $\lambda = m(m+1)$

Scaling: $P_n(1) = 1$, $\int_{-1}^{1} P_m P_n dx = \frac{2}{2n+1}\delta_{mn}$

Zeroes: n zeroes in $[-1, 1]$

Evenness: P_m odd/even if m odd/even

First Four Legendre Polys

$$1,\ x,\ \frac{3x^2-1}{2},\ \frac{5x^3-3x}{2}$$

Generating Function for Legendre

$$\sum_{n=0}^{\infty} P_n(x) r^n = \frac{1}{\sqrt{1-2rx+r^2}}$$

Delta Function:

Sampling

$$\int_a^b f(x)\delta(x-\xi)dx = \begin{cases} f(\xi) & a<\xi<b \\ 0 & \text{o/w} \end{cases}$$

$$g(x)\delta(x) = g(0)\delta(x)$$

Scaling

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$\delta(f(x)) = \sum_{\text{zeros}} \frac{\delta(x-x_i)}{|f'(x_i)|}$$

Dirac Comb Fourier Series

$$\sum \delta(x - 2mL) = \frac{1}{2L}\sum e^{\frac{in\pi x}{L}}$$

δ-function eigenfunction expansion

$$\delta(x-\xi) = \sum w(\xi) Y_n(x) Y_n(\xi) = \sum w(x) Y_n(x) Y_n(\xi)$$

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  - **Flashcards** - *essential*. Active recall. E.g. paper/Anki.
  - **Spreadsheet** - *essential*. Timing past paper questions, keeping track. Conditional formatting...

COURSE	PCT OF TASK COMPLETE	EX SHE		
		1	2	
Revision				
Groups	98%			
Analysis I	100%			
Number Theory	46%			
Differential Equations	97%			
more	50%			

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  - **Coding** - opportunities. Project based learning

ry > disjoint_set_union.py > DSU

```
class DSU():
    # Disjoint Set Union/Union Find
    # n vertices, zero indexed
    # cc = connected components

    def __init__(self, n):
        self.n = n
        self.link = list(range(n))
        self.size = [1]*n

    def find(self, x):
        while (x != self.link[x]):
            x = self.link[x]
        return x

    def same(self, a, b):
        return self.find(a) == self.find(b)

    def unite(self, a, b):
        a = self.find(a)
        b = self.find(b)
        if self.size[a] < self.size[b]:
            a, b = b, a
        self.size[a] += self.size[b]
        self.link[b] = a

    def size(self, x):
        return self.size[self.find(x)]

    def members(self, x):
```

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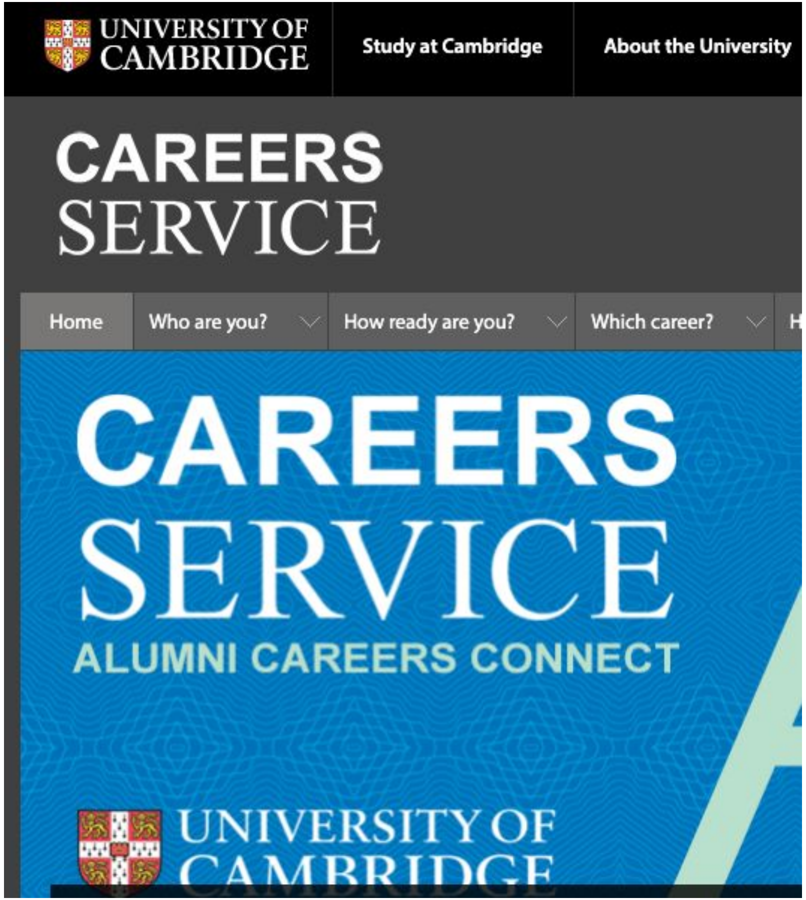
  - **Internships** - get a headstart. Lots to learn from applications - and much to say (but for another time).



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  - **Speak to an older student** - someone in a higher year. Reach out to chat!

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- **Have fun!**

(and maybe make a random Youtube video or two)

Access, log of times spent!

Questions?

- Good luck!
- Slides available here:

  https://michaelng126.wordpress.com/